

Null-Result Detection and Reduction of the State Vector

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January 12, 2011

Abstract

An experiment to investigate the capability of null-result detections (in which what is “registered” is the absence of a particle) to induce a collapse of the state vector via an EPR correlation is proposed, and the conceptual consequences of its possible outcomes are discussed.

I. Introduction

By null-result (NR) detection I mean “detection” in which, instead of registering the presence of a particle, the detector “registers” its absence [1]. In other words, since there is no detector “click,” it is possible to know—at least in ideal situations—the path that the particle has not followed. If there are only two possible paths, we can also infer the path followed by the particle. Naturally, from the habitual standpoint there seems to be nothing mysterious in this. On the other hand, although they involve no irreversible amplification (that is usually associated with the measurement process [2]), it is accepted that NR detections have the same capability of reducing (or collapsing) the quantum state vector as ordinary (O) detections. Here I will show—examining the connection between NR detections [3,4], Einstein-Podolsky-Rosen (EPR) correlations [5], and Bell inequalities [6]—how this could be experimentally verified using two-photon entangled states.

First, let me draw attention to the importance of considering time-like events, a point that has been overlooked in previous discussions on NR detection. As an example, let us consider the following simple experiment. A single photon (from a pair generated via spontaneous parametric down-conversion (SPDC) [7], for instance), impinges on a 50:50 beam splitter. A first detector is placed near the beam splitter to register a reflected photon, and a second one is placed distant from the beam splitter to register a transmitted photon. Whenever the first detector does not click, it is possible to infer that the photon has been transmitted. From a quantum mechanical standpoint, it can be said that the absence of detection induced a reduction of the state vector, “forcing” [8] the photon into the transmitted state. However, the lack of detection at the first detector and the detection at the second are space-like events [9];

therefore, there are an infinite number of Lorentz frames in which the second detector clicks before the first does not click. Hence, it can equally be said that the detection of the second photon induced the collapse of the reflected state. Consequently, this cannot be considered an undisputable NR-detection measurement.

Naturally, the above example also admits a simple classical interpretation: in an ideal situation, if a particle has not been reflected, it has necessarily been transmitted [10]. To avoid this sort of “familiar” explanation, it is important to consider an experiment that violates a Bell inequality, namely one in which an objective (as opposed to subjective) change of probability must necessarily occur.

As has been pointed out [4], the Gedanken experiments discussed by Renninger [11] and Dicke [12] also admit ordinary explanations, so to speak, in which the lack of detection, although giving us information about the state of the system, does not necessarily imply that the system has been forced into this state by this very lack of detection. The same is true when we infer, from the absence of resonance fluorescence, that a quantum jump has occurred [1, 13]. Therefore, it seems fair to say that no unquestionable collapse inducing NR-detection measurement has been performed since the emergence of quantum mechanics, more than eighty years ago. Naturally, NR-detection reduction of the state vector is far from being a trivial fact, and its experimental verification is extremely important for the investigation of the foundations of quantum mechanics. More specifically, it is important to try to establish *when* and *where* the reduction of the state vector takes place (and even if it does indeed takes place), or if the quantum-classical frontier can be arbitrarily established [14], or if indeed “no elementary quantum phenomenon is a phenomenon until it is a registered (observed) phenomenon” [2, 15].

The following example makes it clear that even a very simple situation can pose a tricky question. Let us consider the experiment represented in Fig. 1. Photons ν_1 and ν_2 are generated via SPDC and, independently of the Lorentz frame we use to describe the experiment, ν_1 is always detected before ν_2 . This can be achieved by introducing a detour on the path followed by ν_2 . Assuming, as customary [16], that there is an essential uncertainty about the time of emission, and, consequently, ν_1 and ν_2 are initially totally “delocalized,” how does detector D_1 “know” when it is time to make a “click”? In principle, we can assume that nature is intrinsically stochastic, and the click results from spontaneous fluctuations. However, how does detector D_2 know that it cannot make a click since D_1 has not made a click yet? It seems that even quantum fluctuations can be nonlocally correlated. An unorthodox alternative might be to try to develop some kind of pilot wave interpretation a la de Broglie-Bohm [17], in which photons would not be delocalized. This is a possibility that cannot

be rejected a priori [18].

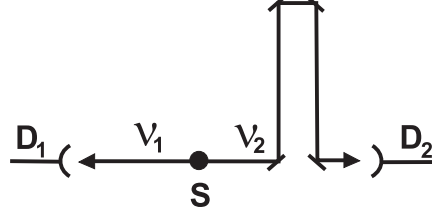


Fig. 1: A thought experiment to disclose intriguing features of quantum mechanics. A source (S) emits, via SPDC, delocalized twin photons (ν_1 and ν_2). In all Lorentz frames ν_1 is detected before ν_2 .

II. The Experiment

To try to clarify these questions, let us then consider the experiment represented in Fig. 2, which is a variant of an experiment performed by Aspect, Grangier, and Roger [19]. A source S generates pairs of photons (ν_1, ν_2) in the state [20]

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|a\rangle |a\rangle + |a_\perp\rangle |a_\perp\rangle), \quad (1)$$

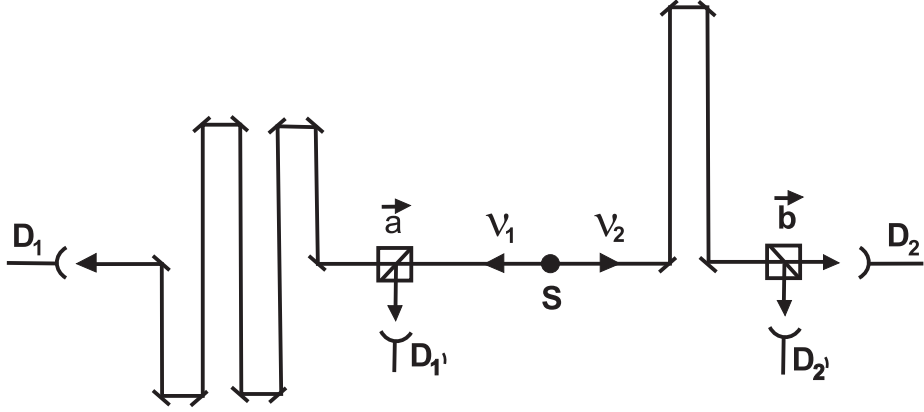


Fig. 2: A proposed experiment to investigate the capability of NR detections to induce a collapse of the state vector: whenever ν_1 is not detected at $D_{1'}$, ν_2 is forced into a well-defined polarization state.

where $|a\rangle$ ($|a_\perp\rangle$) represents a linear polarization state parallel (perpendicular) to \mathbf{a} . ν_1 and ν_2 are sent in opposite directions: ν_1 impinges on a two-channel polarizer oriented parallel to \mathbf{a} , and ν_2 (after following a detour) on a two-channel polarizer oriented parallel to \mathbf{b} . Photons ν_1 that are transmitted have to follow a detour, before impinging on detector D_1 . Photons ν_1 that are reflected impinge on detector $D_{1'}$. Photons ν_2 that are transmitted (reflected) impinge on detector D_2 ($D_{2'}$). The distances are such that, independently of the Lorentz

frame we use to describe the experiment, ν_1 is always detected (or not detected) at detector $D_{1'}$ before ν_2 is detected at D_2 or $D_{2'}$, and ν_2 is always detected at D_2 or $D_{2'}$ before ν_1 is detected at D_1 . Therefore, unless we assume that the future can influence the past (which is questionable), the detection of ν_1 at D_1 cannot force ν_2 into a well-defined polarization state; similarly, the detection of ν_2 at D_2 or $D_{2'}$ cannot force ν_1 into a well-defined polarization state, since ν_1 has already either been or not been detected at $D_{1'}$; the only possibility is the detection (or non-detection) of ν_1 at $D_{1'}$ forcing ν_2 . Now it is possible to know—indisputably, I would say—if the NR detection is indeed capable of reducing the state vector (naturally, we can only be certain that an NR detection has occurred when both photons of the same pair are detected).

If, as generally believed, NR detections are capable of inducing the reduction of the state vector, we must have

$$p(a_{\perp}, b) = p(a, b_{\perp}) = \frac{1}{2} \sin^2(a, b) \quad (2)$$

and

$$p(a_{\perp}, b_{\perp}) = p(a, b) = \frac{1}{2} \cos^2(a, b), \quad (3)$$

where $p(a_{\perp}, b)$ is the probability of detecting ν_1 in a polarization state perpendicular to \mathbf{a} and ν_2 in a polarization state parallel to \mathbf{b} , and so on. On the other hand, if NR detections are incapable of inducing the reduction of the state vector, we must have

$$p'(a_{\perp}, b) = \frac{1}{2} \sin^2(a, b), \quad (4)$$

$$p'(a_{\perp}, b_{\perp}) = \frac{1}{2} \cos^2(a, b), \quad (5)$$

$$p'(a, b) = \frac{1}{2} \alpha(a, b) \quad (6)$$

and

$$p'(a, b_{\perp}) = \frac{1}{2} [1 - \alpha(a, b)], \quad (7)$$

where, in general, $\alpha(a, b) \neq \cos^2(a, b)$, since, when ν_1 is not detected at $D_{1'}$, ν_2 is not forced into a polarization state parallel to \mathbf{a} [21], and the condition $p'(a, b) + p'(a, b_{\perp}) = p'(a) = 1/2$ is fulfilled. It is interesting to observe that

$$p'(b) = p'(a, b) + p'(a_{\perp}, b) = \frac{1}{2} [\alpha(a, b) + \sin^2(a, b)] \quad (8)$$

and

$$p'(b_{\perp}) = p'(a, b_{\perp}) + p'(a_{\perp}, b_{\perp}) = \frac{1}{2} [1 - \alpha(a, b) + \cos^2(a, b)]. \quad (9)$$

Therefore, we may have $p'(b) \neq p'(b_{\perp})$.

To determine the disagreement between the NR-detection reduction of the state vector (NR-detection collapse, for short) and the no-NR-detection collapse approaches, I will introduce a local model [4]. Using (7) and (6), the probabilities will be given by

$$p_L(a_{\perp}, b) = p_L(a, b_{\perp}) = p'(a, b_{\perp}) = \frac{1}{2} [1 - \alpha(a, b)] \quad (10)$$

and

$$p_L(a_\perp, b_\perp) = p_L(a, b) = p'(a, b) = \frac{1}{2}\alpha(a, b). \quad (11)$$

Hence, the correlation function will be

$$\begin{aligned} E_L(a, b) &= p_L(a, b) - p_L(a, b_\perp) \\ &- p_L(a_\perp, b) + p_L(a_\perp, b_\perp) = 2\alpha(a, b) - 1. \end{aligned} \quad (12)$$

Using the CHSHB inequality [22]

$$|E_L(a, b) - E_L(a, b') + E_L(a', b) + E_L(a', b')| \leq 2, \quad (13)$$

choosing $\text{angle}(a, b) = \text{angle}(a', b) = \text{angle}(a', b') = \frac{1}{3}\text{angle}(a, b') = 22.5^\circ$, and taking into account that $E_L(67.5^\circ) = -E_L(22.5^\circ)$, we obtain

$$4E_L(22.5^\circ) \leq 2, \quad (14)$$

which, using (12), leads to

$$\alpha(22.5^\circ) \leq \frac{3}{4}. \quad (15)$$

Using (8) and (9) we obtain

$$p'(b_\perp) - p'(b) = \frac{1}{2} - \alpha + \frac{1}{2}\cos 45^\circ, \quad (16)$$

which, using (15), leads to

$$p'(b_\perp) - p'(b) \geq \frac{\sqrt{2} - 1}{4} \approx 0.1, \quad (17)$$

in disagreement with the quantum mechanical prediction $p(b_\perp) - p(b) = 0$ [23]. Naturally, if there is no correlation, $\alpha = \text{const.} = 1/2$, $p'(a, b) = p'(a, b_\perp) = \text{const.} = 1/4$, and instead of (17) we will have $p'(b_\perp) - p'(b) \approx 0.35$. But, in this case, the greatest disagreement is obtained choosing $\mathbf{a} = \mathbf{b}$, which, using (9) and (8), leads to $p'(b_\perp) = 3/4$ and $p'(b) = 1/4$, and, using (7), to $p'(b, b_\perp) = 1/4 \neq p(b, b_\perp) = 0$.

III. The Real Situation

In a real situation (assuming, for purposes of simplification, that all detectors have the same efficiency, and that the two polarizers are identical), instead of (2) and (3), we have [19,22]

$$p(a, b) = p(a_\perp, b_\perp) = \frac{1}{4}\eta^2 fg [T_+^2 + FT_-^2 \cos 2(a, b)] \quad (18)$$

and

$$p(a, b_\perp) = p(a_\perp, b) = \frac{1}{4}\eta^2 fg [T_+^2 - FT_-^2 \cos 2(a, b)], \quad (19)$$

where η is the detectors' efficiency; f is the probability of the first photon being collected; g is the probability of the second photon being collected when the first has been collected; $T_\pm = T_\parallel \pm T_\perp$, where $T_\parallel(T_\perp)$ is the transmission coefficient for light polarized parallel (perpendicular) to the polarizer's orientation; and F

indicates the amount of correlation between the photons ($0 \leq F \leq 1$, where $F = 1$ corresponds to perfect correlation, and $F = 0$ to no correlation at all). Actually, unlike what has been done in (17), the best procedure (from a practical point of view) is to consider only the coincident detections in which D_1 clicks since when $D_{1'}$ clicks no NR detection occurs. From (18) and (19) we see that

$$p_c(a) = p(a, b) + p(a, b_\perp) = \frac{1}{2}\eta^2 fgT_+^2, \quad (20)$$

where the subscript c indicates that we are only considering situations in which ν_1 and ν_2 are both detected (coincident detections). Therefore,

$$p(b | a) = \frac{p(a, b)}{p_c(a)} = \frac{1}{2} \left[1 + F \frac{T_-^2}{T_+^2} \cos 2(a, b) \right] \quad (21)$$

and

$$p(b_\perp | a) = \frac{p(a, b_\perp)}{p_c(a)} = \frac{1}{2} \left[1 - F \frac{T_-^2}{T_+^2} \cos 2(a, b) \right], \quad (22)$$

where $p(b | a)$ [$p(b_\perp | a)$] is the probability of ν_2 being detected at D_2 [$D_{2'}$] when ν_1 is detected at D_1 . Using (21) and (22) we obtain

$$p(b | a) - p(b_\perp | a) = F \frac{T_-^2}{T_+^2} \cos 2(a, b), \quad (23)$$

which can be written, using only directly observable quantities, as

$$\frac{N(a, b) - N(a, b_\perp)}{N(a, b) + N(a, b_\perp)} = F \frac{T_-^2}{T_+^2} \cos 2(a, b), \quad (24)$$

where $N(a, b)$ [$N(a, b_\perp)$] is the number of coincident detections at D_1 and D_2 [$D_{2'}$]. Hence, using the data from the experiment by Aspect, Grangier, and Roger [19] ($T_\parallel \approx 0.950$, $T_\perp \approx 0.007$, and $F \approx 0.984$), we see that

$$\frac{N(a, b) - N(a, b_\perp)}{N(a, b) + N(a, b_\perp)} \approx 0.696, \quad [\text{angle}(a, b) = 22.5^\circ]. \quad (25)$$

On the other hand, in a real situation, if no NR-detection collapse occurs, instead of (6) and (7) we must have

$$p'(a, b) = \frac{1}{4}\eta^2 fg [T_+^2 + \beta(a, b)] \quad (26)$$

and

$$p'(a, b_\perp) = \frac{1}{4}\eta^2 fg [T_+^2 - \beta(a, b)], \quad (27)$$

where, in general, $\beta(a, b) \neq FT_-^2 \cos 2(a, b)$. Considering an ideal situation, in which all photons are collected and detected, and introducing a local model satisfying the conditions

$$p_L(a, b) = p_L(a_\perp, b_\perp) = \frac{p'(a, b)}{\eta^2 fg T_+^2} \quad (28)$$

and

$$p_L(a, b_\perp) = p_L(a_\perp, b) = \frac{p'(a, b_\perp)}{\eta^2 fg T_+^2}, \quad (29)$$

instead of (12) we obtain

$$E_L = \frac{\beta(a, b)}{T_+^2}, \quad (30)$$

which, using (13), leads to

$$\frac{\beta(a, b)}{T_+^2} \leq \frac{1}{2}, \quad [angle(a, b) = 22.5^\circ]. \quad (31)$$

From (26) and (27) we have

$$p'_c(a) = p'(a, b) + p'(a, b_\perp) = \frac{1}{2}\eta^2 fgT_+^2. \quad (32)$$

Hence,

$$p'(b | a) = \frac{p'(a, b)}{p'_c(a)} = \frac{1}{2} \left[1 + \frac{\beta(a, b)}{T_+^2} \right] \quad (33)$$

and

$$p'(b_\perp | a) = \frac{p'(a, b_\perp)}{p'_c(a)} = \frac{1}{2} \left[1 - \frac{\beta(a, b)}{T_+^2} \right]. \quad (34)$$

Therefore, using (31), we can write

$$p'(b | a) \leq \frac{1}{2} \left(1 + \frac{1}{2} \right) \quad (35)$$

and

$$p'(b_\perp | a) \geq \frac{1}{2} \left(1 - \frac{1}{2} \right), \quad (36)$$

which leads to

$$p'(b | a) - p'(b_\perp | a) \leq \frac{1}{2} \quad (37)$$

and to

$$\frac{N'(a, b) - N'(a, b_\perp)}{N'(a, b) + N'(a, b_\perp)} \leq 0.5, \quad [angle(a, b) = 22.5^\circ], \quad (38)$$

in strong disagreement with (25). Assuming, in Fig. 2, the distance from S to the first polarizer as being equal to 1 meter, we can easily see that the lengths of the detours for ν_2 and ν_1 , respectively, can be of approximately 2 and 4 m, which can easily be accomplished using optical fibers.

IV. Discussion

We see that if (authentic) NR detections do not induce the collapse of the state vector, then the correlation between photons ν_1 and ν_2 may disappear, or at best be reduced to a classical one. This seems to violate the quantum mechanical formalism; however, at least in principle, a small modification of this formalism would be able to incorporate this result. First, we would have to know when there is and when there is no collapse, and then “mechanically” apply the formalism. In other words, expression (1), with its usual physical interpretation, would not always be valid (we would have a similar situation if the expected violations of Bell’s inequalities were not confirmed).

There are two possible results for the experiment that has been discussed and is represented in Fig. 2. The first is that NR-detection collapse does not occur. Although this may sound improbable, the experiment is worth doing, since, as emphasized at the beginning of this paper, NR-detection collapse is far from being a trivial fact. In this case, as stressed in ref. 3 and 4, faster-than-light (FTL) communication would be possible, in principle, provided we are able to establish which photon is *really* detected first. For instance, by monitoring the number of detections on the right-hand side of the experimental apparatus, it would be possible to know whether one of the detectors on the left-hand side has been removed or not [24], since it follows from (17) that $p'(b_\perp) \neq p'(b)$. This is in agreement with Svetlichny's arguments [25] supporting the standpoint according to which a causal theory (i.e., without superluminal communication) implies formal state collapse. In other words, in some situations, if there is no collapse, there must be FTL communication. In the experiment represented in Fig. 2 (with no detours, to have space-like events) this would be a consequence of the coexistence of nonlocal collapse (in the case of O detections) with the absence of nonlocal collapse (in the case of NR detections). Although, at first sight, FTL communication seems inconsistent with special relativity, things may not be so simple. As shown in the appendix, no causal paradox necessarily arises from superluminal signaling.

The second possible result is that, as expected, NR-detection collapse occurs. In this case, from an ontological standpoint, there seem to be two alternatives: (a) adopting a pilot wave interpretation à la de Broglie-Bohm, we can assume that ν_2 is forced into a well-defined polarization state when ν_1 is split in the polarizer into a "full" wave, and an "empty" wave [26]; (b) ascribing an objective meaning (or substance, so to speak) to the probability amplitude, we have to treat O and NR detections on the same footing; that is, each time an O detection (or more generally, a photon absorption) does not occur in one branch of the experiment, the probability amplitudes associated to the other branches are automatically adjusted: instead of photon absorption, we have the absorption (actually, the redistribution) of a probability amplitude, which corresponds to an NR detection, even though we only have an absorber instead of a detector.

The above remarks raise the question about the possibility of quantum mechanics being superseded. The most immediate thought that comes to mind is of a new theory that keeps many essential features of the "old" one. However, Bohr's model of the atom, in which accelerated charges do not necessarily radiate, is a good example of the fact that it is not always wise to become too attached to the prevailing views, even though these views will eventually play an important role in a more elaborate formulation of the new theory [27]. NR detections strongly suggest that no amplification is involved in the collapse of the state vector; therefore, we may conjecture that at a more fundamental level than that ruled by quantum mechanics some as yet unknown processes take place which are responsible for the so called "actualization of potentialities".

V. Conclusion

In this paper it has been assumed that the ordinary (O) detection of a photon of a polarization entangled pair forces its twin into a well-defined polarization state. On the other hand, the consequences of the conjecture that this might not be true for null-result (NR) detections have been investigated. More specif-

ically, it has been pointed out that no noncontroversial collapse inducing NR experiment has been performed so far, and a suggestion to remedy this situation has been presented [28].

Appendix: Superluminal Signaling and Causal Paradox

Let us suppose that behind EPR correlations there is an FTL interaction that can be used for superluminal communication. As we will see, this does not necessarily lead to causal paradoxes, provided that we assume the existence of a preferred frame (an aether, as imagined by Bell) in which the speed of the FTL interaction is always the same, independently of the motion of the “source” (or, equally, of the reference frame in which the experiment is being performed). Let us consider a pair of reference frames, S and S' , in the standard configuration, where S is the preferred frame and S' moves with velocity $v < c$ along the x axis. Assuming that the Lorentz transformations

$$x' = \gamma(x - vt), \quad (\text{a.1})$$

$$t' = \gamma\left(t - \frac{v}{c^2}x\right), \quad (\text{a.2})$$

$$x = \gamma(x' + vt'), \quad (\text{a.3})$$

and

$$t = \gamma\left(t' + \frac{v}{c^2}x'\right), \quad (\text{a.4})$$

connect the S and S' coordinates (with $\gamma = 1/\sqrt{1 - v^2/c^2}$), we derive

$$u' = \frac{u - v}{1 - \frac{vu}{c^2}} \quad (\text{a.5})$$

and

$$u = \frac{u' + v}{1 + \frac{vu'}{c^2}} \quad (\text{a.6})$$

for the velocities.

Let us, initially, see how the causal paradox arises in special relativity (in which there is no preferred frame and S and S' are equivalent). Let the positive quantity $\bar{u} > c$ represent the superluminal signal speed in S . From (a.5), we see that if $u = \bar{u}$, we can choose v so as to have $v\bar{u}/c^2 > 1$, which leads to $u' < 0$ (with $|u'| > c$ but $\neq \bar{u}$). Therefore, in S' the signal propagates backwards. Similarly, from (a.6) we see that, if $u' = -\bar{u}$, we can choose a v that leads to $u > 0$ (with $u > c$ and $\neq \bar{u}$). That is, in S the direction of propagation of the signal is reversed. It is this change of direction when we go from S to S' , and then from S' to S , that is at the origin of the causal paradox. To see this, let us consider a superluminal signal emitted from $x_0 = 0$, at instant $t_0 = 0$, and reaching $x_1 > 0$ at instant t_1 given by

$$t_1 = \frac{x_1}{\bar{u}} \quad (\text{a.7})$$

in S . In S' , the signal is transmitted from $x'_0 = 0$, at instant $t'_0 = 0$, reaching x_1 at instant

$$t'_1 = \gamma\left(t_1 - \frac{v}{c^2}x_1\right) = \gamma\left(1 - \frac{v\bar{u}}{c^2}\right)\frac{x_1}{\bar{u}}, \quad (\text{a.8})$$

according to (a.2). We see that $v\bar{u}/c^2 > 1 \rightarrow t'_1 < 0$. Therefore, in S' the signal reaches x_1 before it is sent from x_0 (actually, the signal is seen to propagate from x_1 to x_0). But this does not yet represent a paradox, since no contradiction (A and $\neg A$, for instance) is occurring. Let us then determine the point x'_1 in S' that coincides with x_1 at the instant at which the signal arrives. Using (a.1) and (a.7), we obtain

$$x'_1 = \gamma \left(x_1 - v \frac{x_1}{\bar{u}} \right) = \gamma \left(1 - \frac{v}{\bar{u}} \right) x_1. \quad (\text{a.9})$$

An observer at x'_1 can then send a return signal with $u' = -\bar{u}$ that will take the time of

$$\delta t' = \frac{x'_1}{\bar{u}} = \gamma \left(1 - \frac{v}{\bar{u}} \right) \frac{x_1}{\bar{u}} \quad (\text{a.10})$$

to arrive at x'_0 . This can lead to a paradox if

$$t'_1 + \delta t' < 0, \quad (\text{a.11})$$

that is, if the return signal reaches the origin of S' before t'_0 , namely before the first signal has been sent. This enables an observer in this region, after receiving the return signal, to inform another observer, at the origin of S , not to send the signal. As a consequence, if the signal is sent, it is possible to send a return signal to impede the emission of the signal. That is, the signal would be sent and not sent at the same time! Let us see the condition v would have to fulfil. From (a.11), (a.10), and (a.8), we obtain

$$\gamma \left(1 - \frac{v\bar{u}}{c^2} \right) \frac{x_1}{\bar{u}} + \gamma \left(1 - \frac{v}{\bar{u}} \right) \frac{x_1}{\bar{u}} < 0, \quad (\text{a.12})$$

which leads to

$$v > \frac{2\bar{u}}{1 + \frac{\bar{u}^2}{c^2}}. \quad (\text{a.13})$$

Since the right-hand side of (a.13) is always smaller than c , it is always possible to find a v that satisfies the above condition; therefore, we would indeed have a paradox.

Now let us see how the existence of a preferred frame in which the superluminal speed is constant does not lead to a causal paradox. Instead of (a.10), we have

$$\delta t' = \frac{x'_1}{-\bar{u}} = -\gamma \left(1 - \frac{v}{\bar{u}} \right) \frac{x_1}{\bar{u}}, \quad (\text{a.14})$$

where the velocity of the return signal (using (a.5)) is

$$\bar{u}' = \frac{-\bar{u} - v}{1 + \frac{v\bar{u}}{c^2}}. \quad (\text{a.15})$$

The condition to have a causal paradox is then

$$\gamma \left(1 - \frac{v\bar{u}}{c^2} \right) \frac{x_1}{\bar{u}} - \gamma \left(1 - \frac{v}{\bar{u}} \right) \frac{x_1}{\bar{u}} < 0, \quad (\text{a.16})$$

where (a.14), (a.11), and (a.8) have been used. From (a.15) and (a.16) we obtain

$$v > c, \quad (\text{a.17})$$

which contradicts our initial assumption that the velocity of reference frame S' is smaller than the velocity of light. As a consequence, there can be no causal paradox.

Naturally, the superluminal interaction that is being considered here breaks the Lorentz symmetry, since the active transformation that would correspond to the passive transformation does not exist; that is, to describe the FTL experiment from a frame that moves with velocity $-\mathbf{v}$ relative to the preferred frame (passive transformation) is not the same as to stay in the preferred frame and describe an FTL experiment in which the experimental apparatus moves with velocity \mathbf{v} (active transformation).

References

- [1] Although the terms “null” measurement (H. Paul, *Introduction to Quantum Theory* (Cambridge University Press, 2008)), “null-result” measurement, and negative-result measurement have been used, they may suggest a different kind of experiment, such as the famous one performed by Michelson and Morley to try to determine the movement of the earth relative to the aether.
- [2] *Quantum Theory and Measurement*, edited by J. A. Wheeler and W. H. Zurek (Princeton University Press, 1983).
- [3] L. C. Ryff, *Phys. Lett. A* **170**, 259 (1992), in which the term “indirect detection” is used. But, in general, indirect detection involves the inference of the existence of something (e.g., elementary particles, dark matter, via Weakly Interacting Massive Particles (WIMPs), et cetera) from the detection of some other thing.
- [4] L. C. Ryff, in *Mysteries, Puzzles, and Paradoxes in Quantum Mechanics*, edited by R. Bonifacio (American Institute of Physics, 1999); L. C. Ryff and C. H. Monken, *Quantum Semiclass. Opt.* **1**, 345 (1999), in which the term “interaction-free measurement” is used. However, this term has become linked to the kind of experiment discussed in A. Elitzur and L. Vaidman, *Found. Phys.* **23**, 987 (1993), which, as will become evident, is actually quite different in spirit from the kind of experiment which is being discussed here. In the Elitzur-Vaidman proposal, whenever a photon is detected in the “forbidden” output port, it is possible to infer the presence of a light absorbing object in one of the arms of a Mach-Zehnder interferometer. Here, whenever a detector does not click, it is possible to infer the path followed by a photon.
- [5] A. Einstein, B. Podolsky, and N. Rosen, *Phys. Rev.* **47**, 777 (1935); D. Bohm, *Quantum Theory* (Prentice-Hall, 1951).
- [6] J. S. Bell, *Speakable and Unspeakable in Quantum Mechanics* (Cambridge University Press, 1989).
- [7] P. G. Kwiat, K. Mattle, H. Weinfurter, A. Zeilinger, A. V. Sergienko, and Y. Shih, *Phys. Rev. Lett.* **75**, 4337 (1995); P. G. Kwiat, E. Waks, A. G. White, I. Appelbaum, and P. H. Eberhard, *Phys. Rev. A* **60** R773 (1999).

- [8] Using a term introduced by Dirac which, in principle, can even be used in interpretations that do not admit an irreversible collapse: P. A. M. Dirac, *The Principles of Quantum Mechanics* (Clarendon Press, 1958).
- [9] It may sound strange to consider the lack of detection as an event; however, taking into account the context, there seems to be no reason for any confusion.
- [10] Actually, the transmitted and reflected probability amplitudes can be recombined by means of a Mach-Zehnder interferometer, and the detectors can be placed at the interferometer output ports, so that interference can be observed. As a result, we don't have, in the true sense of the word, a classical situation. However, as stressed in ref. 4, it admits, in principle, a simple interpretation based on the de Broglie-Bohm pilot-wave approach (D. Bohm and J. B. Hiley, *The Undivided Universe: An Ontological Interpretation of Quantum Theory* (Routledge, 1993)), in which a wave with a photon ("full" wave) follows one arm of the interferometer and a wave without a photon ("empty" wave) follows the other (F. Selleri, in *The Wave Particle Dualism*, edited by S. Diner *et al* (Kluwer Academic Publishers, 1984)). Strictly speaking, for this kind of experiment, that admits a "classical" interpretation, the consideration of time-like events (by introducing a detour on the photon path leading to the distant detector, for example) would not change it into an undisputable NR-detection experiment. Similarly, by simply changing the position of the detectors, placing the first distant and the second near the beam splitter, this does not improve the situation.
- [11] M. Renninger, *Ztschr. Phys.* **158**, 417 (1960). Renninger has been considered to be the first to draw attention to null measurements (ref. 1); however, in an interesting article, M. A. B. Whitaker, *Prog. Quant. Electron.* **24**, 1 (2000), discusses the gedanken experiment suggested in a precursory paper by P. S. Epstein, *Am. J. Phys.* **13**, 127 (1945).
- [12] R. H. Dicke, *Am. J. Phys.* **49**, 925 (1981), in which—as far as I know—the term "interaction-free" measurement has been introduced.
- [13] Home and Whitaker, in D. Home and M. A. B. Whitaker, *J. Phys. A: Math. Gen.* **25**, 2387 (1992), discuss different kinds of negative-result experiments, and argue that they can be explained without the need to invoke the collapse of the wave function.
- [14] J. von Neumann, *Mathematical Foundations of Quantum Mechanics* (Princeton University Press, 1955).
- [15] As an aside, it is important to emphasize that the decoherence approach does not solve the measurement problem. Specifically, it does not explain how, from a realm of potentialities the actual world we observe emerges; or, in other words, how our habitual world comes into being. On this subject: O. Pessoa Jr, "Can the Decoherence Approach Help to Solve the Measurement Problem?", *Synthese*, **113** (1998); S. L. Adler, "Why Decoherence has not Solved the Measurement Problem: A Response to P. W. Anderson", *Stud. Hist. Philos. Mod. Phys.*, **34**, 135

- (2003); M. Schlosshauer, “Decoherence, the measurement problem, and interpretations of quantum mechanics”, *Rev. Mod. Phys.* **76**, 1267 (2004); and G. Bacciagaluppi, “The Role of Decoherence in Quantum Mechanics”, <http://plato.stanford.edu/archives/fall2008/entries/qm-decoherence/>, are worth reading. The conflict between the decoherence approach and the special relativity has been emphasized in L. C. Ryff, *J. Mod. Optics* **48**, 905 (2001).
- [16] J. D. Franson, *Phys. Rev. Lett.* **62**, 2205 (1989).
- [17] D. Bohm and B. J. Hiley, and F. Selleri, in ref. 10, and P. R. Holland, *The Quantum Theory of Motion: An Account of the de Broglie-Bohm Causal Interpretation of Quantum Mechanics* (Cambridge University Press, 1995).
- [18] It might be argued that the experiment described by L. J. Wang, X. Y. Zou, and L. Mandel in *Phys. Rev. Lett.* **66**, 1111 (1991) invalidates the pilot wave idea; however, as explicitly stated in their article, their experiment only tests a version of the pilot wave proposed by Croca, Garuccio, Lepore, and Moreira. Similarly, the experiment described by X. Y. Zou, T. Grayson, L. J. Wang, and L. Mandel in *Phys. Rev. Lett.* **68**, 3667 (1992), as explicitly stated by the authors, only tests a version of the pilot wave proposed by Selleri and Croca. In principle, *nonlocal* versions of the pilot wave interpretation cannot be discarded. The possibility of introducing photon trajectories has also been investigated, for example, in P. Ghose, A. S. Majumdar, S. Guha, and J. Sau, *Phys. Lett. A* **290**, 205 (2001), and A. S. Sanz, M. Davidović, M. Božić, and S. Miret-Artés, *Annals of Phys.* **325**, 763 (2010), and references there in.
- [19] A. Aspect, P. Grangier, and G. Roger, *Phys. Rev. Lett.* **49**, 91 (1982).
- [20] To simplify, I will consider ideal polarization correlations and polarizers. I will also assume that all photon pairs are detected, which is formally equivalent to assuming perfect angular correlations, no losses, and 100% efficient detectors. The real situation is discussed in Section III.
- [21] I am assuming that only ordinary detections are capable of inducing a reduction of the state vector, and here D_1 always clicks after ν_2 has already been detected.
- [22] J. F. Clauser and A. Shimony, *Rep. Prog. Phys.* **41**, 1881 (1978).
- [23] Considering non ideal polarization correlations and polarizers, using the same procedure as in ref. 4 (the only difference is that now only the events in which both photons of the pair are detected are being taken into account), it is possible to show that $p'_c(b_\perp) - p'_c(b) > 0.059$.
- [24] Naturally, to have FTL communication we have to consider space-like events (no detours in Fig. 2) and all ν_2 that are detected, for instance, independently of ν_1 also being detected or not (no classical communication channel to inform on that). We also have to implicitly assume that there is a preferred frame in which one of the photons (ν_1 , in this example) is really detected first. Arguments in favor of a preferred frame have been presented by John Bell in “How to teach special relativity,” in J. S. Bell,

- Speakable and Unspeakable in Quantum Mechanics* (Cambridge University Press, 1989), and in an interview in *The Ghost in the Atom*, eds P. C. W. Davies and J. R. Brown (Cambridge University Press, 1989). In this last book, Bohm explicitly entertains the idea of FTL signaling in an interview. Bohm and Hiley also advocate a preferred frame in ref. 10. The possibility of introducing and identifying a preferred frame is examined in P. Caban and J. Rembieliński, *Phys. Rev. A* **59**, 4187 (1999), in J. Rembieliński and K. A. Smoliński, *Phys. Rev. A* **66**, 052114 (2002), and more recently in J. Rembieliński and K. A. Smoliński, *EPL* **88**, 10005 (2009).
- [25] G. Svetlichny, *Found. Phys.* **33**, 641 (2003). The use of the no signaling condition as an axiom to build the fundamental structure of quantum mechanics has also been discussed in: G. Svetlichny, *Found. Physics* **28**, 131 (1998); G. Svetlichny, *Found. Physics* **30**, 1819 (2000); and C. Simon, V. Bužek, and N. Gisin, *Phys. Rev. Lett.* **87**, 170405 (2001).
- [26] Actually, according to Bohm and Hiley (in ref. 10), in the case of boson fields, we should give up the notion of particle—and, naturally, also of pilot wave—and consider the field variables themselves as fundamental ontological entities. However, it is not clear in this approach how to deal with NR-detection collapse. It is worth mentioning that, as acknowledged by Holland in ref. 17 (p. 481), a detailed treatment of the optical experiments testing Bell’s inequalities has not yet been given in the de Broglie-Bohm causal interpretation. It is also important to emphasize that, strictly speaking, there is no collapse in this interpretation. The same is valid for alternative (a): if the probability amplitudes associated with the transmitted and reflected photon ν_1 were recombined (by changing the configuration of the experiment represented in Fig. 2), ν_1 and ν_2 would become entangled again. That is, although ν_2 is forced into a well-defined polarization state when ν_1 is split in the polarizer, no collapse occurs, since entanglement can be recovered (in this case, ν_2 “loses” the polarization it had got).
- [27] A. Bokulich, *Reexamining The Quantum-Classical Relation* (Cambridge University Press, 2008), for an interesting analysis of the interconnection between quantum and classical theories, including historic aspects.
- [28] To my knowledge, the de Broglie-Bohm causal interpretation does not yet seem to provide a clear answer for this kind of experiment when photons are used, and interpretations that consider the reduction of the state vector in EPR correlations only an epistemic phenomenon are not satisfactory. Likewise, abandoning realism does not seem to be a solution to the nonlocality conundrum, as clearly argued in N. Gisin, “Non-realism: deep thought or a soft option?”, *arXiv: quant-ph/0901.4255*, T. Norsen, *Found. Phys.* **37**, 311 (2007), and F. Laudisa, *Found. Phys.* **38**, 1110 (2008). According to Polkinghorne, “One must acknowledge that a true case of action at a distance is involved, and not merely some gain in additional knowledge. Putting in a learned language, the EPR effect is ontological and not simply epistemological.”, J. Polkinghorne, *Quantum Theory: A Very Short Introduction* (Oxford University Press, 2002), p. 80 (a very lucid and concise explanation of the foundations of quantum mechanics). T. Maudlin, *Quantum Non-Localilty and Relativity* (Blackwell, 2002) is also a good read.